

# Capacitors



# Capacitors

Capacitance

Capacitance of  
parallel plate capacitor

Energy stored in a capacitor

Energy density

Series and parallel combination

Other types of capacitors

# Electric potential and capacitance

## Capacitance

Charge given to a body results in development of potential on it which depends on the geometrical dimensions of the body and the medium.

$$Q \propto V$$

$$\Rightarrow Q = C V$$

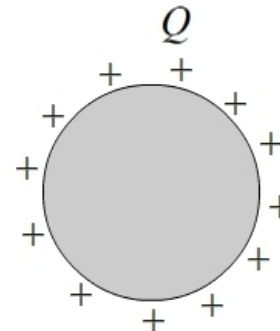
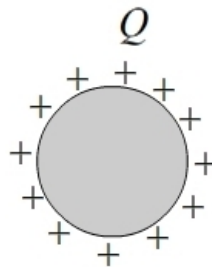
$C$  is the constant of proportionality called capacitance of the body.

Capacitance of a body is defined as the ratio of charge on the body to its potential.

$$C = \frac{Q}{V}$$

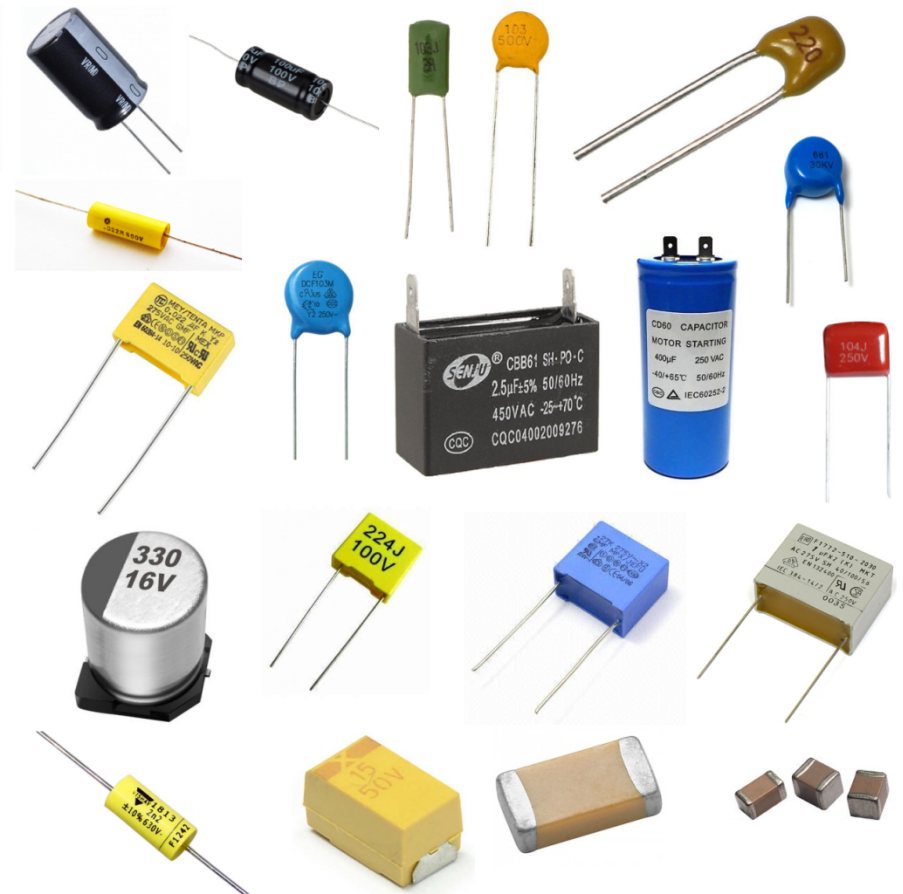
SI unit of capacitance is farad ( F )

High potential  
Less capacitance



Low potential  
More capacitance

## Capacitors in the market



# Electric potential and capacitance

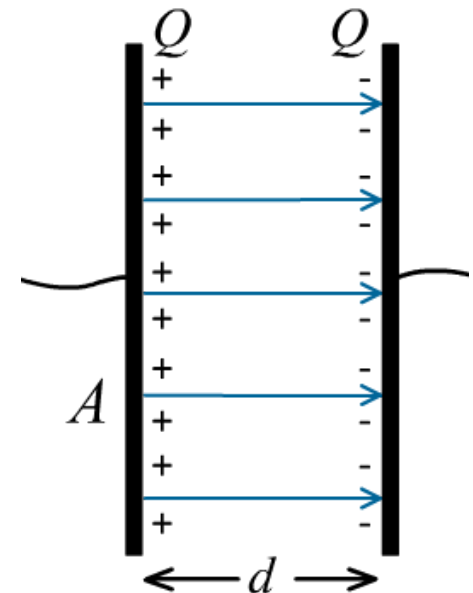
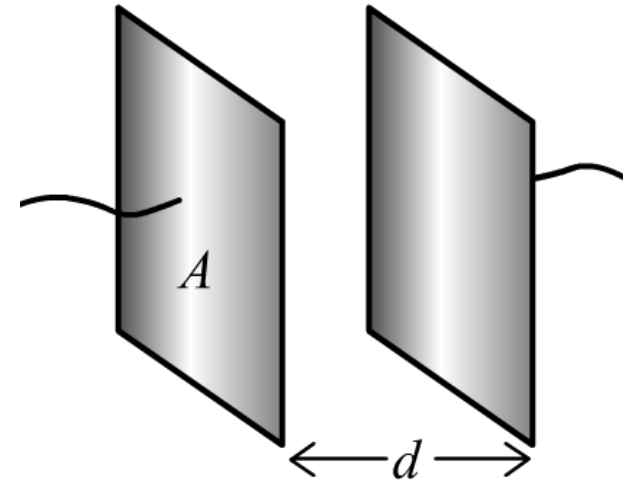
## Parallel plate capacitor ( capacitance )

A parallel plate capacitor is the most common form of capacitor used in the industry.

It is made of two identical parallel plates, each of area  $A$  separated by a distance  $d$ .

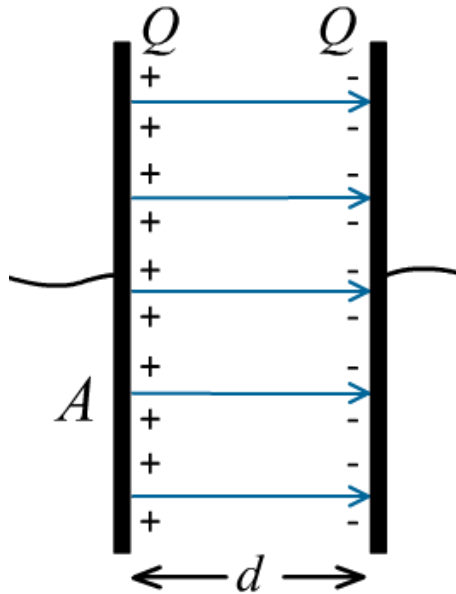
When connected to a cell, the plates acquire equal and opposite charge ( supplied by the cell ).

In a *steady state*, the potential difference across the plates of a single capacitor connected to a cell is equal to the potential difference across the cell.



# Electric potential and capacitance

## Our study of parallel plate capacitor



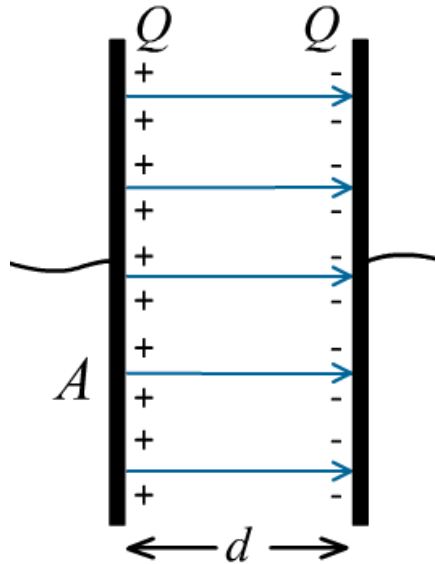
- Capacitance of a parallel plate capacitor
- Energy stored in a capacitor
- Energy density due to electric field energy in a parallel plate capacitor
- Effective capacitance of a series combination of capacitors
- Effective capacitance of a parallel combination of capacitors
- Effect of dielectric on a capacitor
- Growth of a charge in a capacitor connected to a cell
- Decay of charge in a capacitor connected to a resistor



# Electric potential and capacitance

## Capacitance of a parallel plate capacitor

Consider a capacitor with two parallel plates, each of area  $A$  separated by a distance  $d$ . Let the charge on the capacitor be  $Q$ .



Capacitance is given by

$$C = \frac{Q}{V} \quad \text{--- i}$$

Surface charge density of each plate is

$$\sigma = \frac{Q}{A} \quad \text{--- ii}$$

$E$  due to each plate, assumed to be infinite, is given by  $E = \sigma/2\epsilon_0$ . Considering  $E$  due to both the plates we get

$$E = \frac{\sigma}{\epsilon_0} \quad \text{--- iii}$$

$V$  is related to electric field as

$$V = E d$$

Using the expression for  $V$  in eq (iii)

$$\Rightarrow V = \frac{\sigma}{\epsilon_0} d \quad \text{--- iv}$$

Substituting (iv) and (ii) in (i) we get

$$C = \sigma A \frac{\epsilon_0}{\sigma d}$$

$$C = \frac{A \epsilon_0}{d}$$

**Note :** Comparison of  $C$  with a similar expression of  $R$  is worthwhile !

# Electric potential and capacitance

## Energy stored in a parallel plate capacitor

Consider a small amount of charge  $dq$  added to the plates under the existing potential difference of  $V$ . Work done in this case is given by

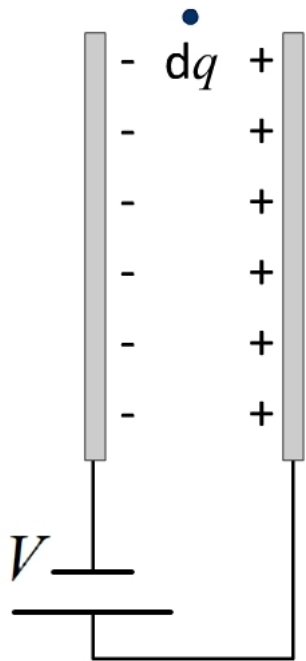
$$dW = dq V$$

Total work done in depositing charge from 0 to  $Q$  is

$$W = \int_0^Q dq V$$

Using the relation

$$q = CV \Rightarrow V = \frac{q}{C}$$



$$W = \int_0^Q dq \frac{q}{C}$$

Capacitance of a capacitor is constant ( it depends only on the dimensions of the capacitor ). Therefore

$$W = \frac{1}{C} \int_0^Q q dq$$

$$W = \frac{1}{C} \left( \frac{Q^2}{2} \right)$$

$$U = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{1}{2} CV^2$$

## Electric potential and capacitance

### Energy stored in the capacitor and energy spent by the cell

Energy stored in the capacitor when it acquires a charge  $Q$  is

$$U = \frac{QV}{2}$$

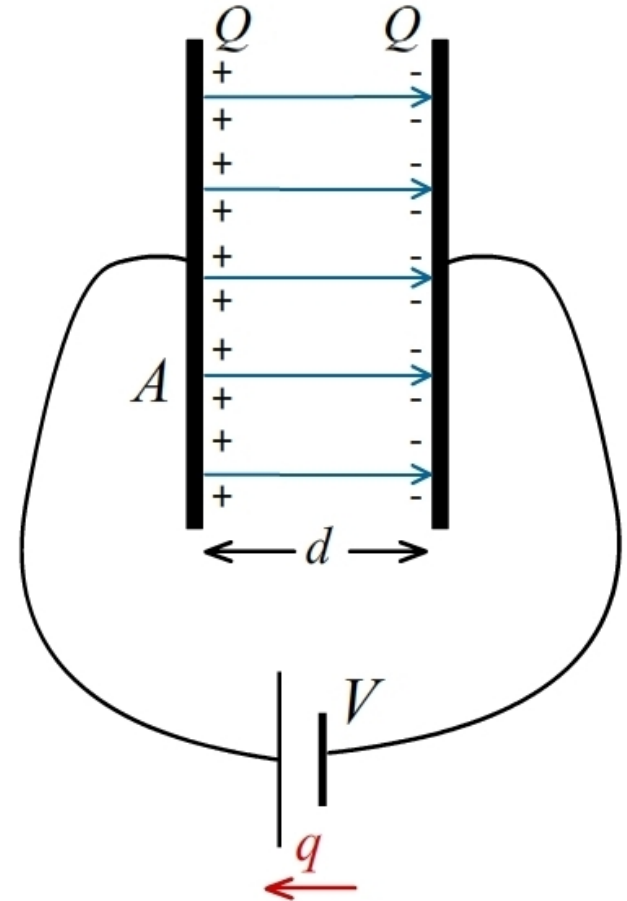
Total work done in transferring charge  $Q$  across the terminals of the cell is

$$W = QV$$

Energy stored is not equal to the energy spent !

Therefore only half the work done by the cell is stored as electrostatic potential energy in the capacitor.

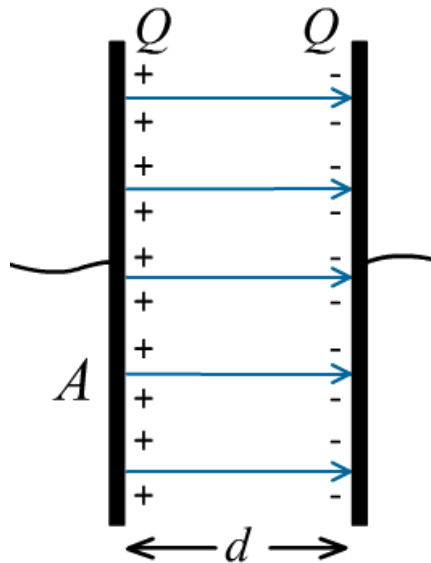
The other half is lost in the form of heat during the process of charging.



# Electric potential and capacitance

## Parallel plate capacitor ( energy density )

Consider a parallel plate capacitor consisting of two parallel plates, each of area  $A$  separated by a distance  $d$  charged to potential  $V$ .



Energy stored in it given by

$$U = \frac{1}{2} CV^2$$

Energy per unit volume is given by

$$\frac{U}{V} = \frac{1}{2} CV^2 \frac{1}{Ad}$$

Using the relations

$$C = \frac{A\epsilon_0}{d} \quad \text{and} \quad V = Ed$$

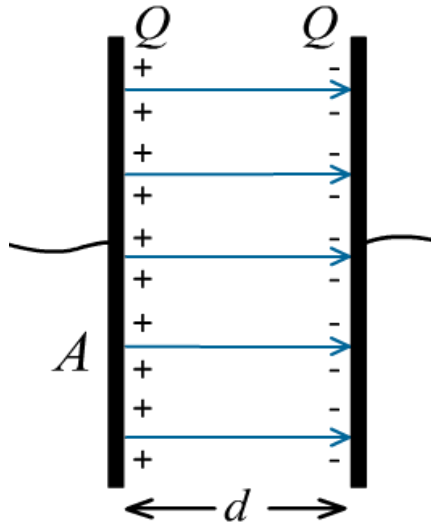
$$\frac{U}{V} = \frac{1}{2} \left( \frac{A\epsilon_0}{d} \right) (E^2 d^2) \frac{1}{Ad}$$

$$\text{energy density} = \frac{1}{2} \epsilon_0 E^2$$

# Electric potential and capacitance

## Force between the plates of a parallel plate capacitor

Consider a parallel plate capacitor of capacitance  $C$  charged to a potential  $V$ . Let the charge on each plate be  $Q$ .



Force acting on charge  $q$  in a region of electric field of intensity  $E$  is given by

$$F = qE$$

Force acting on the total charge  $Q$  is

$$F = QE$$

Using the relation for electric field due to each plate we get

$$F = Q \frac{\sigma}{2\epsilon_0}$$

Using the relation  $\sigma = Q/A$  we get

$$F_{\text{on each plate}} = \frac{Q^2}{2\epsilon_0 A}$$

## Series combination of capacitors ( effective capacitance )

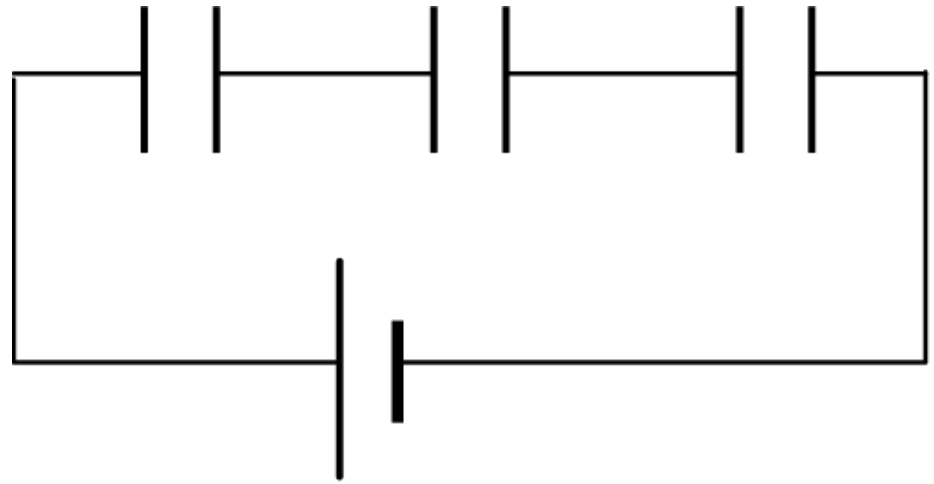
Consider three capacitors of capacitances  $C_1$ ,  $C_2$  and  $C_3$  connected in series as shown in the figure.

Charge (  $Q$  ) on each capacitor is same.

Potential difference (  $V$  ) across each capacitor depends on charge (  $Q$  ) and capacitance (  $C$  ) of the capacitor.

Total potential difference is equal to the potential of the cell, therefore

$$V = V_1 + V_2 + V_3 \quad \text{--- i}$$



Using the relation  $Q = CV$  we get  $V = \frac{Q}{C}$

Substituting this in eq (i) we get

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

### Parallel combination of capacitors ( effective capacitance )

Consider three capacitors of capacitances  $C_1$ ,  $C_2$  and  $C_3$  connected in parallel as shown in the figure.

Potential difference (  $V$  ) across each capacitor is same.

Charge (  $Q$  ) on each capacitor depends on capacitance (  $C$  ) of the capacitor.

Total charge supplied by cell is therefore given by

$$Q = Q_1 + Q_2 + Q_3$$

Using the relation  $Q = CV$  we get

$$CV = C_1V + C_2V + C_3V$$

$$C = C_1 + C_2 + C_3$$

